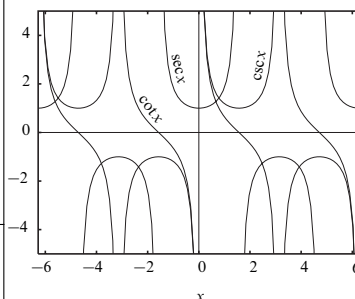
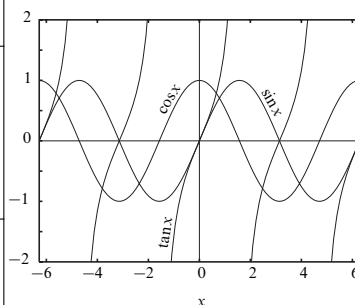


2.5 Trigonometric and hyperbolic formulas

Trigonometric relationships

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	(2.171)
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	(2.172)
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	(2.173)
$\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$	(2.174)
$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$	(2.175)
$\sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$	(2.176)
$\cos^2 A + \sin^2 A = 1$	(2.177)
$\sec^2 A - \tan^2 A = 1$	(2.178)
$\csc^2 A - \cot^2 A = 1$	(2.179)
$\sin 2A = 2 \sin A \cos A$	(2.180)
$\cos 2A = \cos^2 A - \sin^2 A$	(2.181)
$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$	(2.182)
$\sin 3A = 3 \sin A - 4 \sin^3 A$	(2.183)
$\cos 3A = 4 \cos^3 A - 3 \cos A$	(2.184)
$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$	(2.185)
$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$	(2.186)
$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$	(2.187)
$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$	(2.188)
$\cos^2 A = \frac{1}{2} (1 + \cos 2A)$	(2.189)
$\sin^2 A = \frac{1}{2} (1 - \cos 2A)$	(2.190)
$\cos^3 A = \frac{1}{4} (3 \cos A + \cos 3A)$	(2.191)
$\sin^3 A = \frac{1}{4} (3 \sin A - \sin 3A)$	(2.192)



Hyperbolic relationships^a

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y \quad (2.193)$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y \quad (2.194)$$

$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y} \quad (2.195)$$

$$\cosh x \cosh y = \frac{1}{2} [\cosh(x+y) + \cosh(x-y)] \quad (2.196)$$

$$\sinh x \cosh y = \frac{1}{2} [\sinh(x+y) + \sinh(x-y)] \quad (2.197)$$

$$\sinh x \sinh y = \frac{1}{2} [\cosh(x+y) - \cosh(x-y)] \quad (2.198)$$

$$\cosh^2 x - \sinh^2 x = 1 \quad (2.199)$$

$$\operatorname{sech}^2 x + \tanh^2 x = 1 \quad (2.200)$$

$$\coth^2 x - \operatorname{csch}^2 x = 1 \quad (2.201)$$

$$\sinh 2x = 2 \sinh x \cosh x \quad (2.202)$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x \quad (2.203)$$

$$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x} \quad (2.204)$$

$$\sinh 3x = 3 \sinh x + 4 \sinh^3 x \quad (2.205)$$

$$\cosh 3x = 4 \cosh^3 x - 3 \cosh x \quad (2.206)$$

$$\sinh x + \sinh y = 2 \sinh \frac{x+y}{2} \cosh \frac{x-y}{2} \quad (2.207)$$

$$\sinh x - \sinh y = 2 \cosh \frac{x+y}{2} \sinh \frac{x-y}{2} \quad (2.208)$$

$$\cosh x + \cosh y = 2 \cosh \frac{x+y}{2} \cosh \frac{x-y}{2} \quad (2.209)$$

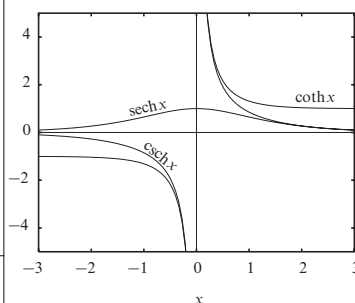
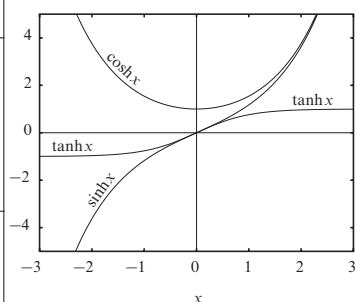
$$\cosh x - \cosh y = 2 \sinh \frac{x+y}{2} \sinh \frac{x-y}{2} \quad (2.210)$$

$$\cosh^2 x = \frac{1}{2} (\cosh 2x + 1) \quad (2.211)$$

$$\sinh^2 x = \frac{1}{2} (\cosh 2x - 1) \quad (2.212)$$

$$\cosh^3 x = \frac{1}{4} (3 \cosh x + \cosh 3x) \quad (2.213)$$

$$\sinh^3 x = \frac{1}{4} (\sinh 3x - 3 \sinh x) \quad (2.214)$$



^aThese can be derived from trigonometric relationships by using the substitutions $\cos x \mapsto \cosh x$ and $\sin x \mapsto i \sinh x$.



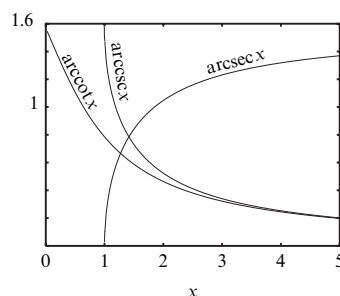
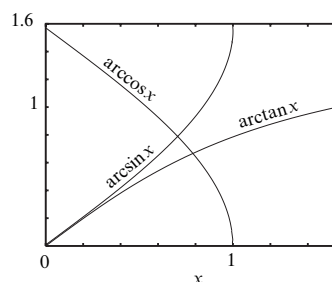
Trigonometric and hyperbolic definitions

de Moivre's theorem	$(\cos x + i \sin x)^n = e^{inx} = \cos nx + i \sin nx$	(2.215)
$\cos x = \frac{1}{2} (e^{ix} + e^{-ix})$	(2.216)	$\cosh x = \frac{1}{2} (e^x + e^{-x})$ (2.217)
$\sin x = \frac{1}{2i} (e^{ix} - e^{-ix})$	(2.218)	$\sinh x = \frac{1}{2} (e^x - e^{-x})$ (2.219)
$\tan x = \frac{\sin x}{\cos x}$	(2.220)	$\tanh x = \frac{\sinh x}{\cosh x}$ (2.221)
$\cos ix = \cosh x$	(2.222)	$\cosh ix = \cos x$ (2.223)
$\sin ix = i \sinh x$	(2.224)	$\sinh ix = i \sin x$ (2.225)
$\cot x = (\tan x)^{-1}$	(2.226)	$\coth x = (\tanh x)^{-1}$ (2.227)
$\sec x = (\cos x)^{-1}$	(2.228)	$\operatorname{sech} x = (\cosh x)^{-1}$ (2.229)
$\csc x = (\sin x)^{-1}$	(2.230)	$\operatorname{csch} x = (\sinh x)^{-1}$ (2.231)

Inverse trigonometric functions^a

$\arcsin x = \arctan \left[\frac{x}{(1-x^2)^{1/2}} \right]$	(2.232)
$\arccos x = \arctan \left[\frac{(1-x^2)^{1/2}}{x} \right]$	(2.233)
$\operatorname{arccsc} x = \arctan \left[\frac{1}{(x^2-1)^{1/2}} \right]$	(2.234)
$\operatorname{arcsec} x = \arctan \left[(x^2-1)^{1/2} \right]$	(2.235)
$\operatorname{arccot} x = \arctan \left(\frac{1}{x} \right)$	(2.236)
$\arccos x = \frac{\pi}{2} - \arcsin x$	(2.237)

^aValid in the angle range $0 \leq \theta \leq \pi/2$. Note that $\arcsin x \equiv \sin^{-1} x$ etc.



Inverse hyperbolic functions

$$\operatorname{arsinh} x \equiv \sinh^{-1} x = \ln \left[x + (x^2 + 1)^{1/2} \right] \quad (2.238) \quad \text{for all } x$$

$$\operatorname{arcosh} x \equiv \cosh^{-1} x = \ln \left[x + (x^2 - 1)^{1/2} \right] \quad (2.239) \quad x \geq 1$$

$$\operatorname{artanh} x \equiv \tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (2.240) \quad |x| < 1$$

$$\operatorname{arcoth} x \equiv \coth^{-1} x = \frac{1}{2} \ln \left(\frac{x+1}{x-1} \right) \quad (2.241) \quad |x| > 1$$

$$\operatorname{arsech} x \equiv \operatorname{sech}^{-1} x = \ln \left[\frac{1}{x} + \frac{(1-x^2)^{1/2}}{x} \right] \quad (2.242) \quad 0 < x \leq 1$$

$$\operatorname{arcsch} x \equiv \operatorname{csch}^{-1} x = \ln \left[\frac{1}{x} + \frac{(1+x^2)^{1/2}}{x} \right] \quad (2.243) \quad x \neq 0$$

